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Space–Time from a Conserved State Vector

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Abstract

This paper develops a foundational theory in which the geometry of spacetime and the dynamics of matter emerge from the evolution of a conserved real state vector, Ψ^μ, in an abstract four-dimensional internal space endowed with a Minkowski metric $\eta_{\mu\nu}$. The theory is constructed from two core axioms: the strict conservation of the state vector's Minkowski norm and the condition of anti-parallelism between its temporal and spatial components. We derive the minimal and covariant action principle consistent with these axioms, which takes the form of a worldline action for a relativistic particle, $S = -m c \int \sqrt{-\eta_{\mu\nu} d\Psi^\mu d\Psi^\nu}$. We demonstrate that the equations of motion describe a Lorentz-rotation of Ψ^μ, with its components $\Psi^\mu \equiv (c t, x^i)$ directly identifiable as physical spacetime coordinates. This identification recovers standard relativistic mechanics, with mass m reinterpreted as the frequency of the state vector's internal oscillation. The framework provides a unified geometric interpretation where physical time, space, motion, and mass are seen as derived, phenomenological aspects of a more fundamental, conserved dynamics in state space. The formulation suggests a natural pathway toward a field-theoretic generalization where the spacetime metric emerges as an induced quantity from the gradients of the state vector field.

Keywords: Emergent Spacetime, Conserved State Vector, Norm Conservation, Anti-Parallelism, Worldline Action, Geometric Mass, Lorentz Rotation.

From a State Vector to an Emergent Metric Field

The Fundamental Variable and Kinematic Constraint

The construction begins by postulating a fundamental object: a four-component real state vector $\Psi^\mu(\lambda)$, where the index $\mu = 0, 1, 2, 3$. The component Ψ^0 is identified as a temporal variable T, while the components Ψ^i (for i=1,2,3) are identified as spatial variables S^i. The parameter λ serves as a bookkeeping parameter for evolution, not an a priori physical time. The fundamental kinematic constraint is the conservation of the "norm" of this vector with respect to the Minkowski metric $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$. This reflects the core axioms of equal magnitudes and anti-parallelism for stable dynamical entities (Fong et al., 2016). The conserved quantity is:

$$Q = \eta_{\mu\nu} \Psi^\mu \Psi^\nu = -(\Psi^0)^2 + (\Psi^1)^2 + (\Psi^2)^2 + (\Psi^3)^2 = \text{constant}.$$

For a real-valued vector, Q can be positive, negative, or zero. We focus on the non-degenerate case $Q \neq 0$, which defines a hyperboloid in the state space. This conservation law is the foundational symmetry from which dynamics will be derived.

Minimal and Covariant Lagrangian Dynamics

To promote the state vector to a dynamical entity, we seek an action principle. The demand for minimalism and covariance leads naturally to a Lagrangian that is proportional to the "square" of the first derivative with respect to λ, contracted with the fixed Minkowski metric. This is analogous to the action for a relativistic point particle, but here the coordinates are the state vector components themselves (Rovelli, 2004). The minimal action is:

$$S[\Psi] = (1/2) \int d\lambda\, \eta_{\mu\nu} (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda).$$

The resulting equation of motion is simply the wave equation in the parameter λ:

$$d^2\Psi^\mu/d\lambda^2 = 0.$$

The solutions are straight lines in the state vector space: $\Psi^\mu(\lambda) = a^\mu \lambda + b^\mu$, where a^μ and b^μ are constants satisfying $\eta_{\mu\nu} a^\mu a^\nu = \text{const}$. While this theory is covariant and minimal, it is too simplistic—it describes a non-interacting, structureless entity. The Minkowski metric $\eta_{\mu\nu}$ here is a fixed, absolute background, not an emergent property. The state vector merely moves through a pre-existing geometric arena, which contradicts the goal of deriving spacetime from the relational properties of the state vector itself.

Introducing Self-Interaction and the Emergent Metric

The key step is to replace the fixed background metric $\eta_{\mu\nu}$ with a dynamical object that depends on the state vector. This embodies the idea that the geometry of the space in which Ψ^μ evolves should be determined by Ψ^μ itself, creating a feedback loop. We introduce a symmetric tensor $g_{\mu\nu}[\Psi]$ which is a functional of the state vector. The minimal covariant action that generalizes the previous one becomes:

$$S[\Psi] = (1/2) \int d\lambda \, g_{\mu\nu}[\Psi] \, (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda).$$

The tensor $g_{\mu\nu}$ is not an independent field; it is defined as a specific function of Ψ^μ to enforce the core axioms. A natural ansatz, inspired by the structure of induced metrics in embedding theories (Guven, 2004), is:

$$g_{\mu\nu}[\Psi] = \alpha \, \eta_{\mu\nu} + \beta \, (\partial\Phi/\partial\Psi^\mu)(\partial\Phi/\partial\Psi^\nu),$$

where α and β are constants, and Φ is a scalar potential function constructed from Ψ. To reflect the conservation law Q = constant, the obvious choice is $\Phi \equiv Q = \eta_{\rho\sigma} \Psi^\rho \Psi^\sigma$. This yields:

$$g_{\mu\nu}[\Psi] = \alpha \, \eta_{\mu\nu} + 4\beta \, \eta_{\mu\rho} \, \eta_{\nu\sigma} \, \Psi^\rho \Psi^\sigma.$$

The second term is a dyadic product $\Psi_\mu \Psi_\nu$, where indices are lowered with η. This metric now depends quadratically on the state vector. The action is now nonlinear:

$$S[\Psi] = (1/2) \int d\lambda \, [\, \alpha \, \eta_{\mu\nu} \dot{\Psi}^\mu \dot{\Psi}^\nu + 4\beta \, (\eta_{\mu\nu} \Psi^\mu \dot{\Psi}^\nu)^2 \,],$$

where the dot denotes a $d/d\lambda$. The second term couples the “velocity” $\dot{\Psi}^\mu$ to the “position” Ψ^μ, introducing a self-interaction. The Euler-Lagrange equations derived from this action are more complex than the wave equation. They describe the autodynamic evolution of a state vector whose effective kinematic space is curved by its own amplitude. The conserved quantity Q now plays the role of a potential shaping the emergent geometry $g_{\mu\nu}$. In this picture, the Minkowski metric $\eta_{\mu\nu}$ retains its role only as an internal symmetry template defining the conserved form Q; the physical metric governing evolution is $g_{\mu\nu}[\Psi]$.

Toward a Full Field Theory: The State Vector as a Field

The final step is to transition from a single state vector evolving in λ to a field defined over a continuum. This is achieved by promoting $\Psi^\mu(\lambda)$ to a field $\Psi^\mu(X^\alpha)$, where X^α are four new abstract coordinates. The parameter λ is absorbed into this manifold. The conservation law becomes a local field constraint: $\eta_{\mu\nu} \Psi^\mu \Psi^\nu$ = constant at each point X. The derivative $d/d\lambda$ is replaced by a partial derivative $\partial_\alpha \equiv \partial/\partial X^\alpha$.

The minimal covariant action in this setting is a sigma-model-like action (Padmanabhan, 2010):

$$S[\Psi] = (1/2) \int d^4X \, \sqrt{-g} \, g^{\alpha\beta} \, G_{\mu\nu}[\Psi] \, \partial_\alpha \Psi^\mu \, \partial_\beta \Psi^\nu,$$

where $g^{\alpha\beta}$ is now the emergent spacetime metric, and $G_{\mu\nu}[\Psi]$ is the metric on the field space (target space) of Ψ. Crucially, $g_{\alpha\beta}$ must itself be constructed from the field Ψ^μ and its derivatives to close the system. A self-consistent closure can be postulated by identifying the spacetime metric with the induced metric on a hypersurface defined by the field configuration:

$$g_{\alpha\beta}(X) = \eta_{\mu\nu} \, \partial_\alpha \Psi^\mu \, \partial_\beta \Psi^\nu.$$

This is a direct generalization of the earlier ansatz for $g\{\mu\nu\}[\Psi]$. Here, $g\{\alpha\beta\}$ measures the "stretch" of the field Ψ^μ in the internal Minkowski space as it maps onto the coordinate manifold X^α. The action then becomes purely a function of the field Ψ:

$$S[\Psi] = (1/2) \int d^4X \sqrt{(-\det(\eta_{\mu\nu} \partial_\alpha \Psi^\mu \partial\beta \Psi^\nu))}\, \eta^{\rho\sigma} \eta\{\mu\nu\} \partial_\rho \Psi^\mu \partial\sigma \Psi^\nu.$$

This is a nonlinear field theory for four scalar fields $\Psi^\mu(X)$ with a Born-Infeld-type structure due to the determinant factor. The equations of motion for $\Psi^\mu(X)$, together with the definition of $g\{\alpha\beta\}$ as the induced metric, create a coupled system where the spacetime geometry and the state field co-determine each other. In this formulation, spacetime points are not fundamental; they are labels for distinguishable configurations of the Ψ^μ field. The causal structure, distances, and durations are all derived from the relational behavior of this field, fulfilling the program of deriving spacetime from a conserved state vector.

Outlook

The progression outlined—from a conserved vector in a fixed metric, to an autodynamic vector with an emergent metric, and finally to a self-consistent field theory—demonstrates a viable pathway for constructing spacetime from simpler algebraic axioms. The core conservation law, rooted in the ideas of dynamical balance, naturally seeds the emergence of a pseudo-Riemannian structure. The final field-theoretic action bears resemblance to theories of emergent gravity from condensates or brane-world models (Volovik, 2009), but with a distinct starting point in the algebra of a state vector. Future work must address quantization, the inclusion of matter degrees of freedom, and the derivation of the Einstein-Hilbert action as an effective limit.

The Geometrical Invariant: From Axioms to Worldline Action

Physical Postulates and their Geometrical Interpretation

The proposed framework is built upon two fundamental, physically motivated axioms for a closed, stable dynamical system represented by a real four-component state vector Ψ^μ (where $\mu = 0,1,2,3$):

1. **Conservation of Magnitude (Norm)**: The "size" of the state vector, as measured by a Minkowski norm, is a constant of motion. This reflects the idea of a system with a fixed total "capacity" or energy-momentum-like invariant. In the context of field theory, such conserved quadratic forms often underlie stability and unitary evolution (Weinberg, 1995).

2. **Dynamics as a Rotation (Lorentz Transformation)**: The internal evolution of the system corresponds to a continuous transformation of the state vector that preserves its norm. In the space spanned by Ψ^μ, the most general such transformations are Lorentz

transformations, SO(1,3). This axiom posits that all possible dynamical histories of the system are connected by paths that are "rotations" in this internal Minkowski space. This concept of dynamics as a flow on a group manifold is a cornerstone of many foundational approaches (Rovelli, 1991).

These axioms have an immediate and powerful geometrical consequence. If the magnitude $\sqrt{|\eta_{\mu\nu} \Psi^\mu \Psi^\nu|}$ is constant, and the dynamics is a continuous norm-preserving transformation, then the evolution of the state vector traces a curve on a hyperboloid embedded in the four-dimensional state space. For a timelike norm ($\eta_{\mu\nu} \Psi^\mu \Psi^\nu < 0$), this is a de Sitter-like hyperboloid; for a spacelike norm, it is a hyperbolic sheet. The parameter λ labels points along this worldline in the state space, not in physical spacetime.

The Minimal Worldline Action

Guided by the principle of minimality and the requirement of covariance under internal Lorentz transformations, we construct the action. In differential geometry, the simplest invariant quantity associated with a curve is its length. Therefore, the most natural action for the evolution of $\Psi^\mu(\lambda)$ on the constraint hyperboloid is proportional to the "length" of its worldline in the state space, measured with the internal Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$. This yields:

$$S[\Psi] = -\alpha \int d\lambda \sqrt{ - \eta_{\mu\nu} (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda) }$$

where α is a positive constant with dimensions of action. The negative sign inside the square root and the overall minus sign ensure a real action for timelike trajectories ($d\Psi^\mu/d\lambda$) in the state space, drawing a direct formal analogy to the proper time of a relativistic particle.

However, the square root form, while geometrically transparent, is inconvenient for quantization and for examining symmetries. A classically equivalent, simpler form is obtained by using a quadratic action, analogous to the Polyakov action in string theory or the einbein formulation of point particle mechanics (Brink, Di Vecchia, & Howe, 1976). We instead postulate the action:

$$S[\Psi] = (1/2) \int d\lambda \left[e^{-1}(\lambda)\, \eta_{\mu\nu} (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda) - e(\lambda) M^2 \right].$$

Here, $e(\lambda)$ is an auxiliary field (an "einbein") that can be considered as a square root of the induced metric on the one-dimensional worldline. The constant M^2 is related to the fixed norm Q from the axioms. Varying the action with respect to $e(\lambda)$ gives the constraint equation:

$$e^2(\lambda) = - (1/M^2)\, \eta_{\mu\nu} (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda).$$

Substituting this back into the quadratic action recovers the original square-root form, identifying $\alpha = M$. This demonstrates the equivalence. The conserved quantity from the equations of motion is precisely:

$$Q = \eta_{\mu\nu} \Psi^\mu \Psi^\nu = \text{constant},$$

which is a direct manifestation of Axiom 1. The equation of motion obtained by varying Ψ^μ is:

$$(d/d\lambda)\left[e^{-1}(\lambda) (d\Psi^\mu/d\lambda) \right] = 0.$$

This describes a geodesic motion on the flat state space, but subject to the constraint Q = constant. The solutions are of the form $\Psi^\mu(\lambda) = A^\mu \cosh(\omega\lambda) + B^\mu \sinh(\omega\lambda)$ for a timelike Q, representing a hyperbolic "rotation" or boost in the state space—a continuous Lorentz transformation. This directly embodies Axiom 2: the dynamical evolution is indeed a flow generated by the Lorentz group.

Symmetries and the Emergence of a Prespacetime

This simple worldline model already exhibits rich symmetry structures that hint at the origins of spacetime:

1. **Reparametrization Invariance**: The action is invariant under arbitrary monotonic transformations of the parameter $\lambda \to \lambda'(\lambda)$. This is a fundamental gauge symmetry, indicating that λ itself has no physical meaning; only the ordered sequence of states matters. This symmetry is the one-dimensional analog of diffeomorphism invariance in general relativity.

2. **Global Internal Lorentz Invariance**: The action is manifestly invariant under constant SO(1,3) transformations of the state vector: $\Psi^\mu \to \Lambda^\mu{}_\nu \Psi^\nu$. This is the symmetry of the fixed metric $\eta_{\mu\nu}$.

3. **Conservation Laws**: The reparametrization invariance leads to a constraint (the "Hamiltonian constraint" upon canonical quantization), which is the expression of the fixed norm. The global Lorentz invariance leads to conserved "angular momentum" charges in state space: $L^{\mu\nu} = \Psi^\mu (d\Psi^\nu/d\lambda) - \Psi^\nu (d\Psi^\mu/d\lambda)$. These charges are the generators of the internal Lorentz transformations.

At this stage, we have a "prespacetime" theory. The coordinates X^μ of a physical spacetime are not yet present. The fundamental entities are abstract states Ψ^μ and their evolution parameter λ. However, the mathematical structure is identical to that of a single relativistic particle propagating in a fictitious four-dimensional Minkowski space. This is a crucial point: the machinery of special relativity (worldlines, proper time, Lorentz invariance) appears here not as a description of physical spacetime, but as a description of the intrinsic dynamics of an abstract state vector obeying simple conservation axioms.

The Need for Generalization: From Worldline to Field

The free worldline model, while foundational, is insufficient to generate a dynamical spacetime geometry. It describes only a single, non-interacting degree of freedom. Physical spacetime is characterized by local relationships, fields, and many degrees of freedom interacting at different "places."

The logical next step, therefore, is to promote the worldline theory to a field theory. This involves two key conceptual leaps:

1. **Promoting the State Vector to a Field**: Instead of a single worldline $\Psi^\mu(\lambda)$, we consider a continuous infinity of them, or equivalently, a field $\Psi^\mu(x^a)$ where x^a are four new, initially meaningless manifold coordinates. The parameter λ is absorbed, and evolution is now in this new coordinate space.

2. **Making the Metric Dynamical**: The fixed internal metric $\eta_{\mu\nu}$ must be replaced by a dynamical metric $g_{\mu\nu}$ that depends on the field Ψ^μ. This is the step where the internal symmetry of the state space begins to dictate the geometry of the coordinate manifold x^a. The constraint of conserved norm must now be applied locally, leading to a nonlinear theory.

The worldline action of this section serves as the generating seed for these constructions. Its symmetries—reparametrization invariance and Lorentz invariance—will be elevated to become the guiding principles for building a field-theoretic action in the next stage of development, moving decisively from the mechanics of a single state to the dynamics of a field from which spacetime itself can emerge.

The Core Lagrangian: Worldline Dynamics in State Space

Constructing the Minimal Lagrangian

Following the axiomatic foundation of a conserved state vector norm and dynamics as rotation, we now construct the minimal dynamical theory. Let the fundamental variable be the four-component real state vector $\Psi^\mu(\lambda) = (T(\lambda), S^i(\lambda))$, where $i=1,2,3$. The parameter λ orders states but is not identified with physical time a priori. The fixed internal metric is $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$.

The axioms dictate two strict constraints on any allowed Lagrangian L:

1. It must yield the conservation of the Minkowski norm: $(d/d\lambda)(\eta_{\mu\nu} \Psi^\mu \Psi^\nu) = 0$.

2. It must be invariant under global internal Lorentz transformations $\Psi^\mu \to \Lambda^\mu{}_\nu \Psi^\nu$, reflecting the rotational symmetry of the state space.

The simplest scalar quantity that can be constructed from the first derivatives of the state vector is the "kinetic term" $\eta_{\mu\nu} (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda)$. This term is manifestly Lorentz invariant. To enforce the conservation of the norm as an equation of motion (rather than as a rigid constraint), we must ensure the Lagrangian possesses a specific symmetry. The action $S = \int L \, d\lambda$ must be invariant under transformations that leave the norm invariant. The kinetic term alone, however, does not generally conserve Ψ^2 unless specific boundary conditions or additional constraints are applied.

The direct route to a theory where the norm is dynamically conserved is to use the Lagrangian for a free relativistic particle, but reinterpreted in state space. This yields the core, minimal Lagrangian:

$$L_0 = -\alpha \sqrt{ - \eta_{\mu\nu} \dot{\Psi}^\mu \dot{\Psi}^\nu } = -\alpha \sqrt{ \dot{T}^2 - \dot{S}^i \dot{S}_i }.$$

Here, α is a positive constant with dimensions of action, and the dot denotes differentiation with respect to λ. The expression under the square root must be positive for a timelike trajectory in state space, hence the minus sign inside the radical. This Lagrangian is precisely analogous to the Lagrangian for a massive relativistic particle moving in a 4D Minkowski spacetime (Landau & Lifshitz, 1975), but here the "spacetime" is the abstract space of the state vector components (T, S^i).

Physical Interpretation: Dynamics as Internal Rotation

The physical meaning of L_0 is profound and distinct from conventional particle mechanics. The system described by $\Psi^\mu(\lambda)$ does not move through physical space or time. Instead, it "rotates" or evolves within its own internal state space. The temporal component $T(\lambda)$ and the spatial components $S^i(\lambda)$ are treated democratically as coordinates on this internal manifold.

To see this explicitly, consider the canonical momenta derived from L_0:

$$p_\mu = \partial L_0/\partial \dot{\Psi}^\mu = \alpha (\eta_{\mu\nu} \dot{\Psi}^\nu) / \sqrt{ - \eta_{\rho\sigma} \dot{\Psi}^\rho \dot{\Psi}^\sigma }.$$

These momenta satisfy the primary constraint:

$$\eta^{\mu\nu} p_\mu p_\nu + \alpha^2 = 0.$$

This is a mass-shell condition in the momentum space of the state vector. The Hamiltonian, constructed via Legendre transformation, vanishes identically due to reparametrization invariance in λ—a hallmark of theories describing evolution in terms of worldline geometry (Henneaux & Teitelboim, 1992).

The equations of motion are:

$$(d/d\lambda)[\dot{\Psi}_\mu / \sqrt{ - \dot{\Psi}^2 }] = 0.$$

The general solution is a straight line in the state space when parameterized by the analogue of proper time. However, when we fix the parameter λ arbitrarily, the evolution appears as a hyperbolic trajectory. For instance, a simple solution conserving a timelike norm $Q = -R^2$ is:

$$T(\lambda) = R \sinh(\omega\lambda),\ S^1(\lambda) = R \cosh(\omega\lambda),\ S^2 = S^3 = 0.$$

This describes a continuous Lorentz boost in the (T, S^1) plane of state space. The state vector sweeps out a hyperbola, its tip tracing a path of constant "distance" from the origin. The dynamics is pure internal reorientation—a boost. Another solution could be a rotation in a purely spatial plane (S^1, S^2), conserving a spacelike norm. All such solutions are symmetry transformations of the internal Minkowski space, confirming that the fundamental dynamics is indeed a rotation (a Lorentz transformation) in the space of states, as postulated. This internal evolution is the precursor to what we will later interpret as physical motion and time flow.

Symmetry and the Conserved Norm

The Lagrangian L_0 possesses the key symmetry that enforces the conservation of the state vector's norm. While it is not globally translationally invariant in Ψ-space (which would lead to a trivial theory), its specific square-root form, combined with reparametrization invariance, yields the conserved quantity we seek. Using the equations of motion, one can directly show that:

$$(d/d\lambda)(\eta_{\mu\nu} \Psi^\mu \Psi^\nu) = 0.$$

Thus, the dynamical path dictated by L_0 automatically keeps the state vector on a fixed hyperboloid in state space. The constant value of this norm, Q, becomes a fundamental parameter characterizing different "sectors" of the theory—analogous to mass in particle physics.

The presence of the square root in L_0 makes the quantization and analysis of interactions non-trivial. Therefore, it is often advantageous to use an equivalent, classically isomorphic formulation (Polyakov, 1981) that is quadratic in derivatives:

$$L_0' = (1/(2e(\lambda)))\, \eta_{\mu\nu} \Psi'^\mu \Psi'^\nu - (e(\lambda)\, \alpha^2)/2.$$

Here, $e(\lambda)$ is an auxiliary einbein field. Variation with respect to $e(\lambda)$ yields its equation of motion:

$$e(\lambda) = \sqrt{ - \eta_{\mu\nu} \Psi'^\mu \Psi'^\nu } / \alpha.$$

Substituting this back into L_0' recovers the original L_0 up to a total derivative. This quadratic Lagrangian makes the internal Lorentz symmetry and the constraint structure more transparent for canonical analysis. The constraint from $e(\lambda)$ becomes:

$$\eta_{\mu\nu} \Psi'^\mu \Psi'^\nu + e^2(\lambda)\, \alpha^2 = 0,$$

which is the classical precursor to the mass-shell condition.

Limitations and the Path to a Field Theory

The Lagrangian L_0, while elegant and axiomatically faithful, describes a single, isolated worldline in state space. It represents a zero-dimensional object—a "point" in the state manifold—evolving along its trajectory. This is insufficient to describe a universe with multiple entities, interactions, and local degrees of freedom that could be identified with a physical spacetime geometry.

The key limitation is the absence of any notion of locality or field-ness. All information is contained in the single vector $\Psi^\mu(\lambda)$. There is no concept here of something happening "here" versus "there." To progress from this mechanical model to a theory from which spacetime can emerge, we must promote the state vector from a function of a single parameter λ to a field over a continuum. That is, we must consider $\Psi^\mu(x^a)$, where x^a are four new coordinates that will eventually acquire the meaning of spacetime coordinates through the dynamics of Ψ itself.

This promotion requires a radical step: the internal metric $\eta_{\mu\nu}$, which is fixed in L_0, must become dependent on the field Ψ^μ and its derivatives with respect to the new coordinates x^a. The Lagrangian will then become a functional of $\Psi^\mu(x)$ and its gradients, $L[\Psi, \partial\Psi]$. The conserved norm condition will become a local constraint, and the "worldline" action $S = \int L_0 \, d\lambda$ will generalize to a "spacetime" action $S = \int L[\Psi, \partial\Psi] \, d^4x$. The core structure of L_0—its reliance on the Minkowskian kinetic term and its link to conservation—will serve as the guiding template for this construction, ensuring the axioms of norm conservation and rotational dynamics are embedded at the heart of the emerging field theory.

Implementing Anti-Parallelism: The Constraint of Spatial Alignment

The Anti-Parallelism Axiom as a Linear Constraint

The foundational axioms of the theory are two-fold: the conservation of the state vector's Minkowski norm and the condition of anti-parallelism between the spatial and temporal components. While the first axiom leads to the kinetic structure of the Lagrangian L_0, the second axiom imposes a critical geometric constraint on the configuration of the state vector itself. This axiom posits that, for a fundamental, stable dynamical unit, the spatial part of the state vector S is strictly anti-aligned with the temporal part T, up to a universal constant of proportionality. Mathematically, this is expressed as:

$$S = -c \, \hat{n} \, T$$

where S is the spatial vector (S^1, S^2, S^3), T is the temporal component Ψ^0, c is a fundamental constant with dimensions of velocity, and $\hat{n}$ is a fixed unit vector in the internal spatial state space. The fixity of $\hat{n}$ is crucial; it breaks the full internal SO(3) rotational symmetry of the spatial state subspace down to an axial symmetry around $\hat{n}$. This condition implies that the state vector $\Psi^\mu = (T, S)$ is not free to point in any direction of the internal (1+3)-dimensional Minkowski space. Instead, its spatial orientation is rigidly locked to its temporal magnitude. Physically, this can be interpreted as a definition of a preferred "rest frame" or alignment axis within the internal state space, a concept with parallels in theories of spontaneous symmetry breaking (Nambu & Jona-Lasinio, 1961). The constant c, which will be identified with the speed of light, sets the scale between temporal and spatial magnitudes, ensuring they are commensurate.

Lagrange Multiplier Formulation

The Lagrangian formalism provides a powerful and systematic method for incorporating such holonomic constraints: the method of Lagrange multipliers (Lanczos, 1970). We start from the free, reparametrization-invariant Lagrangian L_0 derived from the norm conservation axiom. The anti-parallelism condition, being a configurational constraint at each value of the evolution parameter λ, is enforced by adding a term linear in a new set of dynamical fields—the Lagrange multipliers.

The constraint is a vector equation in the internal spatial space. Therefore, it requires a spatial vector of Lagrange multipliers, which we denote $\Lambda_i(\lambda)$. The total constrained Lagrangian becomes:

$$L_{total} = L_0 + L_{constraint}$$

where the constraint term is:

$$L_{constraint} = \Lambda_i (S^i + c\, n^i T).$$

Here, n^i are the fixed components of the unit vector $\hat{n}$. The multipliers $\Lambda_i(\lambda)$ are auxiliary fields; they carry no kinetic term of their own and their equations of motion will precisely yield the constraint we wish to enforce.

The action principle is now $S = \int d\lambda\, L_{total}$. Variation with respect to the primary fields yields modified equations of motion. Crucially, variation with respect to each Lagrange multiplier Λ_i gives:

$$\delta S/\delta\Lambda_i = 0 \rightarrow S^i + c\, n^i T = 0.$$

This is exactly the anti-parallelism axiom, recovered as an equation of motion. The multipliers Λ_i themselves are determined by varying the action with respect to the original state vector components T and S^i. These variations produce equations that dictate the force-like role of the constraints, ensuring the dynamics are consistent with the rigid alignment.

Physical Consequences and Symmetry Reduction

Imposing this constraint has profound implications for the dynamics and the physical interpretation of the theory.

1. **Reduction of Degrees of Freedom**: The unconstrained state vector Ψ^μ has four independent components. The three independent equations $S^i = -c\, n^i T$ reduce the number of independent configuration space variables to one. Essentially, the entire state vector is determined by a single scalar degree of freedom, $T(\lambda)$, with $S(\lambda)$ slaved to it. The dynamical content simplifies dramatically.

2. **Emergence of a Preferred Axis**: The fixed vector $\hat{n}$ selects a distinguished direction in the internal state space. This breaks the isotropy of the internal spatial sector. While the full theory with L_0 alone was invariant under internal spatial rotations (part of the Lorentz group SO(1,3)), the constrained theory is only invariant under rotations around the $\hat{n}$ axis. This is a clear case of symmetry breaking, where the dynamics (via the constraint) picks out a specific vacuum direction. In cosmological terms, this is reminiscent of a vector field with a fixed vacuum expectation value, which can influence the structure of spacetime (Ackerman et al., 2007).

3. **Modified Equations of Motion**: Substituting the constraint $S^i = -c\, n^i T$ directly into the original Lagrangian L_0 yields an effective Lagrangian for the single remaining dynamical variable. Assuming $\hat{n}$ is a timelike unit vector in the full state space (so its internal Minkowski norm is -1), the kinetic term becomes:

   $$L_0 = -\alpha \sqrt{(- (\dot{T}^2 - \dot{S}\cdot\dot{S}))} = -\alpha \sqrt{(-\dot{T}^2 (1 - c^2))}.$$

For this to be real and non-degenerate for non-zero $\dot{T}$, we must have $c^2 = 1$. This identifies the constant c as the fundamental "state-space speed," which will correspond to the invariant speed of light in the emergent spacetime. With c=1, the Lagrangian simplifies to $L_{0\_eff} = -\alpha \sqrt{(0)}$, which is singular. This indicates that the naive substitution is too crude; the full dynamics must be derived from the constrained action before solving the multiplier equations. The correct procedure shows that the multipliers Λ_i enforce a form of motion where the evolution of T and S is consistent with both the norm conservation and the linear constraint, leading to a consistent, albeit constrained, geodesic motion in state space.

From Rigid Constraint to Dynamical Field

The formulation with a fixed $\hat{n}$ and rigid constraint, while implementing the axiom precisely, is overly strict for a fundamental theory aimed at deriving spacetime. A fixed, global $\hat{n}$ is non-dynamical and seems artificial. The more physical and fruitful approach is to promote the anti-parallelism condition from a rigid constraint to a dynamical principle.

This can be achieved by allowing the direction $\hat{n}$ to become a function of the evolution parameter, $\hat{n}(\lambda)$, or, in the subsequent field-theoretic extension, a field $\hat{n}(x)$. The constraint is then not on the state vector relative to an absolute background direction, but rather as a condition linking T and S to an internal frame field that itself may evolve. In this more sophisticated picture, the Lagrange multiplier term becomes:

$$L_{constraint} = \Lambda_i (S^i + c\, n^i(\lambda)\, T) + L_{dyn}[\hat{n}],$$

where $L_{dyn}[\hat{n}]$ is a new kinetic term for the field $\hat{n}$, allowing it to have its own dynamics. The anti-parallelism condition now defines the spatial state S relative to a dynamical internal director $\hat{n}$. In the limit where the dynamics of $\hat{n}$ are frozen, we recover the rigid constraint.

This step is critical for the transition to a field theory. In the continuum limit, where Ψ^μ becomes a field $\Psi^\mu(x^a)$ and $\hat{n}$ becomes a spacetime vector field $n^a(x)$, the constraint may take a form like $S^\mu(x) \propto n^\mu(x)\, T(x)$. This intertwines the state field with what will become a tetrad or a velocity field in the emergent spacetime, a key feature in formulations of emergent gravity and analogue models (Barcelo, Liberati, & Visser, 2005). The Lagrange multiplier method thus provides the formal bridge from a simple, rigid axiom to a rich, dynamical geometric structure.

An Equivalent Formulation: The Invariant Norm Lagrangian

From Constrained Dynamics to the Covariant Oscillator

The previous sections developed the theory using a Lagrangian whose square-root structure directly enforced reparametrization invariance and, via constraints, the anti-parallelism condition. There exists, however, a more elegant and algebraically transparent formulation that encodes the axiom of norm conservation not as a consequence of a constraint, but as the central dynamical principle itself. This formulation draws a powerful analogy with one of the most fundamental systems in physics: the harmonic oscillator.

Consider the following Lagrangian, quadratic in both the "velocity" Ψ'^{μ} and the "position" Ψ^{μ}:

$L = (1/2)\, \eta_{\mu\nu}\, \Psi'^{\mu}\, \Psi'^{\nu} - (1/2)\, \omega^2\, \eta_{\mu\nu}\, \Psi^{\mu}\, \Psi^{\nu}$.

This is precisely the Lagrangian for a four-dimensional isotropic harmonic oscillator in a Minkowski space (Bohm, 1993). The first term is the kinetic energy in the state space, and the second term is a potential energy that is quadratic in the state vector's Minkowski norm. The constant ω has dimensions of inverse time (in λ units) and sets the oscillator's characteristic frequency.

This formulation is equivalent in its physical content to the constrained worldline theory, but it expresses the axioms in a different, arguably more fundamental, language. The conservation of the norm is no longer an add-on; it is built into the very structure of the potential. The potential $V = (1/2)\, \omega^2\, \eta_{\mu\nu}\, \Psi^{\mu}\, \Psi^{\nu}$ attains its minimum not at a point, but on the entire null cone $\eta_{\mu\nu}\, \Psi^{\mu}\, \Psi^{\nu} = 0$. For a non-zero ω, the dynamics will naturally conserve the specific value of this norm determined by initial conditions, much like energy conservation in a standard oscillator.

Equations of Motion: Space and Time in Counterphase

The Euler-Lagrange equations derived from this Lagrangian are the equations of a Minkowski-space oscillator:

$d^2\Psi^{\mu}/d\lambda^2 + \omega^2\, \Psi^{\mu} = 0$.

These are four decoupled harmonic oscillator equations, but with a critical pseudo-Euclidean signature. Their general solution is:

$\Psi^{\mu}(\lambda) = A^{\mu} \cos(\omega\lambda) + B^{\mu} \sin(\omega\lambda)$,

where A^{μ} and B^{μ} are constant amplitude vectors.

Now, let us impose the physical interpretation. Recall that $\Psi^{\mu} = (T, S^i)$. The temporal component $T(\lambda)$ and the spatial components $S^i(\lambda)$ are all on equal footing as solutions to the same oscillator equation. Crucially, there is no a priori requirement for them to oscillate in phase. The anti-parallelism axiom can be naturally realized by selecting specific solutions where the spatial and temporal oscillations are in precise counterphase. For instance, a fundamental solution satisfying $S = -c \hat{n} T$ (with $c=1$) is:

$T(\lambda) = R \cos(\omega\lambda)$
$S^1(\lambda) = -R \sin(\omega\lambda)$, $S^2 = S^3 = 0$ (for $\hat{n}$ along the x-axis).

This solution yields $\eta_{\mu\nu} \Psi^\mu \Psi^\nu = -R^2(\cos^2(\omega\lambda) - \sin^2(\omega\lambda)) = -R^2 \cos(2\omega\lambda)$, which is not constant. However, a solution that does conserve the norm is:

$T(\lambda) = R \cos(\omega\lambda)$
$S^1(\lambda) = R \sin(\omega\lambda)$.

Here, $\eta_{\mu\nu} \Psi^\mu \Psi^\nu = -R^2$ (constant). In this case, the spatial and temporal components are in quadrature (90° out of phase), not anti-phase. The rigid anti-parallelism condition of Section 4 is thus a specific, limiting case of the more general dynamics allowed by the oscillator Lagrangian. It corresponds to a particular alignment and phase relationship between the four component oscillators. This demonstrates that the oscillator formulation is more general; the specific constraint $S = -\hat{n} T$ can be viewed as a dynamical attractor or a specific solution within this broader class (Guendelman, 1999).

Symmetries and the Emergence of Mass

The Lagrangian $L = (1/2) \eta_{\mu\nu} \Psi'^\mu \Psi'^\nu - (1/2) \omega^2 \eta_{\mu\nu} \Psi^\mu \Psi^\nu$ possesses a rich symmetry structure:

1. **Global Internal Lorentz Invariance**: It is manifestly invariant under $\Psi^\mu \to \Lambda^\mu{}_\nu \Psi^\nu$.
2. **λ-Translation Invariance**: This leads to conservation of the Hamiltonian (energy in λ).
3. **SO(4) or SO(3,1) Symmetry of the Potential**: The potential term has the same pseudo-rotation symmetry as the kinetic term.

The canonical Hamiltonian derived from this Lagrangian is:

$H = (1/2) \eta^{\mu\nu} \pi_\mu \pi_\nu + (1/2) \omega^2 \eta_{\mu\nu} \Psi^\mu \Psi^\nu$,

where $\pi_\mu = \partial L/\partial \Psi'^\mu = \eta_{\mu\nu} \Psi'^\nu$ is the canonical momentum.

This Hamiltonian is strikingly similar to that of a relativistic particle. In fact, if we interpret the evolution parameter λ as proper time, the constraint H = constant can be rewritten as:

$\eta^{\mu\nu} \pi_\mu \pi_\nu = \text{constant} - \omega^2 \eta_{\mu\nu} \Psi^\mu \Psi^\nu$.

Upon quantization, where $\pi_\mu \to -i\hbar\, \partial/\partial\Psi^\mu$, this would yield a Klein-Gordon-like equation in the state space, not in spacetime:

$$[\eta^{\mu\nu} \partial_\mu \partial_\nu + (\omega^2/\hbar^2) \eta_{\mu\nu} \Psi^\mu \Psi^\nu] \Phi(\Psi) = \text{const.}\ \Phi(\Psi).$$

Here, the constant ω^2 plays the role of a mass parameter. Specifically, if we identify $m^2 = \hbar^2 \omega^2 / c^4$ (reintroducing c for clarity), then ω sets the scale of inertia in the state space. This is a profound result: the frequency of oscillation in the abstract state space is directly related to the mass of the entity described by the state vector. A stationary, massive object in physical spacetime corresponds to a high-frequency cyclical evolution in its internal state space. This resonates with the concept of "zitterbewegung" or inherent oscillatory motion attributed to elementary particles in some interpretations of relativistic quantum mechanics (Barut & Zanghi, 1984).

The Oscillator as a Bridge to Field Theory

The harmonic oscillator formulation provides the most direct pathway to a full field theory, which is the ultimate goal of deriving spacetime. The logic is as follows:

1. **Discretization to a Lattice**: Imagine not a single state vector $\Psi^\mu(\lambda)$, but a lattice or network of them, labeled by a discrete index k: $\Psi^\mu_k(\lambda)$. Each node is an independent Minkowski oscillator.

2. **Introduction of Nearest-Neighbor Coupling**: To introduce locality and interaction, we add a coupling term between neighboring oscillators. The simplest covariant coupling is:
   $L_{coupling} = -(\kappa/2) \Sigma_{\langle k,l \rangle} \eta_{\mu\nu} (\Psi^\mu_k - \Psi^\mu_l)(\Psi^\nu_k - \Psi^\nu_l)$,
   where κ is a coupling constant and $\langle k,l \rangle$ denotes neighboring nodes.

3. **Continuum Limit**: In the limit where the lattice spacing becomes infinitesimal, the discrete index k is replaced by continuous coordinates x^a. The field $\Psi^\mu(\lambda)$ becomes a spacetime field $\Psi^\mu(x^a, \lambda)$. The sum over neighbors becomes an integral over spatial gradients:
   $L \to \int d^3x\, [(1/2)(\partial_\lambda \Psi^\mu)(\partial_\lambda \Psi_\mu) - (\kappa/2) (\partial_i \Psi^\mu)(\partial_i \Psi_\mu) - (1/2)\, \omega^2\, \Psi^\mu \Psi_\mu]$.

4. **Absorption of λ**: Finally, to make the theory covariant in the emergent 4D manifold, the parameter λ must be absorbed. This is achieved by reinterpreting the gradient terms to include derivatives with respect to a new, fourth coordinate, promoting λ to x^0. The Lagrangian density then becomes a true field-theoretic one, governing a field $\Psi^\mu(x^\nu)$ in an emergent spacetime:
   $\mathcal{L} = (1/2)\, g^{\alpha\beta}(x)\, \partial_\alpha \Psi^\mu\, \partial_\beta \Psi_\mu - (1/2)\, \omega^2\, \Psi^\mu \Psi_\mu$,
   where $g^{\alpha\beta}$ is the emergent inverse metric.

Thus, the simple covariant oscillator is not just an equivalent model; it is the natural seed from which a local, interacting field theory—and hence a dynamical spacetime geometry—can grow through the principles of discretization, coupling, and continuum limit.

Euler–Lagrange Dynamics: Phase-Locked Oscillations of Space and Time

Derivation of the Covariant Oscillator Equation

From the Lagrangian formalism presented in Section 5, we derive the fundamental equations of motion for the state vector $\Psi^\mu(\lambda)$. The Lagrangian density for the covariant harmonic oscillator is:

$$L = (1/2)\, \eta_{\mu\nu} (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda) - (1/2)\, \omega^2 \eta_{\mu\nu} \Psi^\mu \Psi^\nu.$$

Applying the Euler–Lagrange equation,

$$d/d\lambda\, (\partial L/\partial(d\Psi^\mu/d\lambda)) - \partial L/\partial\Psi^\mu = 0,$$

yields the equation of motion for each component μ:

$$d^2\Psi^\mu/d\lambda^2 + \omega^2 \Psi^\mu = 0.$$

This is a set of four decoupled harmonic oscillator equations, one for each component of the state vector (T, S^1, S^2, S^3). Crucially, the signature of the Minkowski metric $\eta_{\mu\nu}$ does not affect the form of this differential equation; it enters through the conserved quantities and the norm constraint. The parameter ω, with dimensions of frequency, is the fundamental constant of the theory, setting the scale for all dynamical evolution in the parameter λ. This equation is the core dynamical law, describing how the state vector rotates and oscillates within its internal Minkowski space.

General Solution and the Orthogonality Condition

The general solution to this second-order linear differential equation is a superposition of sine and cosine functions:

$$\Psi^\mu(\lambda) = A^\mu \cos(\omega\lambda) + B^\mu \sin(\omega\lambda),$$

where A^μ and B^μ are constant four-vectors representing the amplitudes of the oscillatory modes. This solution describes an elliptical trajectory in the 4D state space projected onto each (A^μ, B^μ) plane.

However, not all choices of A^μ and B^μ are dynamically admissible. The solution must satisfy the conservation of the Minkowski norm $Q = \eta_{\mu\nu} \Psi^\mu \Psi^\nu$, which is a constant of motion derived from the symmetry of the Lagrangian. Substituting the general solution into Q gives:

$$Q = \eta_{\mu\nu} [A^\mu \cos(\omega\lambda) + B^\mu \sin(\omega\lambda)][A^\nu \cos(\omega\lambda) + B^\nu \sin(\omega\lambda)]$$
$$= \eta_{\mu\nu} A^\mu A^\nu \cos^2(\omega\lambda) + \eta_{\mu\nu} B^\mu B^\nu \sin^2(\omega\lambda) + 2\, \eta_{\mu\nu} A^\mu B^\nu \cos(\omega\lambda) \sin(\omega\lambda).$$

For Q to be constant for all values of λ, the coefficients of the time-dependent terms must vanish. This imposes two independent conditions on the amplitude vectors:

1. $\eta_{\mu\nu} A^\mu A^\nu = \eta_{\mu\nu} B^\mu B^\nu = Q_0$, (equal magnitudes)
2. $\eta_{\mu\nu} A^\mu B^\nu = 0$. (orthogonality in the Minkowski sense)

The second condition, $\eta_{\mu\nu} A^\mu B^\nu = 0$, is of paramount physical importance. It states that the amplitude vectors A and B must be orthogonal with respect to the Minkowski metric. In the internal state space, this is not a Euclidean orthogonality but a relativistic one. This condition is the mathematical embodiment of the phase relationship between the oscillatory components of the state vector.

Physical Interpretation: Space-Time Counterphase Oscillation

Let us analyze the implications of the orthogonality condition $\eta_{\mu\nu} A^\mu B^\nu = 0$. Writing the vectors in component form, $A^\mu = (A^0, A)$ and $B^\mu = (B^0, B)$, where A and B are 3-vectors, the condition expands to:

$-A^0 B^0 + A \cdot B = 0$, or equivalently, $A \cdot B = A^0 B^0$.

Now, consider a simple, fundamental mode that realizes the anti-parallelism axiom of Section 4. We choose coordinates such that the oscillation is confined to the (Ψ^0, Ψ^1) plane. Let the amplitude vectors be:
$A^\mu = (R, 0, 0, 0)$ and $B^\mu = (0, R, 0, 0)$.

These vectors satisfy $\eta_{\mu\nu} A^\mu B^\nu = 0$ trivially. The resulting state vector evolution is:

$T(\lambda) = \Psi^0(\lambda) = R \cos(\omega\lambda)$,
$S^1(\lambda) = \Psi^1(\lambda) = R \sin(\omega\lambda)$,
$S^2 = S^3 = 0$.

In this solution, the temporal component T(λ) and the primary spatial component $S^1(\lambda)$ oscillate in quadrature—they are 90 degrees out of phase. When T is at a maximum or minimum ($\cos(\omega\lambda) = \pm 1$), S^1 is zero ($\sin(\omega\lambda) = 0$). Conversely, when S^1 is at an extremum, T is zero. Their oscillations are perfectly phase-shifted.

To achieve strict anti-parallelism ($S = -\hat{n}\, T$) as an instantaneous condition, we would need a solution like $T(\lambda) = R \cos(\omega\lambda)$ and $S^1(\lambda) = -R \cos(\omega\lambda)$. This would correspond to $A^\mu = (R, 0, 0, 0)$ and $B^\mu = (0, -R, 0, 0)$, which does not satisfy $\eta_{\mu\nu} A^\mu B^\nu = 0$ (since $A \cdot B = 0$ but $A^0 B^0 = 0$, so the condition holds only if $R=0$). Therefore, the rigid, instantaneous anti-parallelism is not a solution of the free oscillator equations. Instead, the orthogonal amplitude condition enforces a dynamical relationship where space and time components oscillate out of phase, with their instantaneous magnitudes related such that the Minkowski norm is preserved. The rigid anti-parallelism can be interpreted as an averaged or dressed state of this more fundamental oscillatory motion, a point of view reminiscent of the "stitching" of classical trajectories from underlying oscillatory dynamics (Elze, 2012).

Conserved Charges and Lorentz Algebra

The Lagrangian possesses global internal Lorentz invariance. According to Noether's theorem, this implies the existence of conserved charges. For the solution $\Psi^\mu(\lambda) = A^\mu \cos(\omega\lambda) + B^\mu \sin(\omega\lambda)$, the conserved angular momentum tensor in state space is:

$J^{\mu\nu} = \Psi^\mu (d\Psi^\nu/d\lambda) - \Psi^\nu (d\Psi^\mu/d\lambda)$.

Substituting the solution and averaging over a period of oscillation (or using the amplitude vectors directly), one finds that the conserved charges are proportional to:

$J^{\mu\nu} \propto A^\mu B^\nu - A^\nu B^\mu$.

The orthogonality condition $\eta_{\mu\nu} A^\mu B^\nu = 0$ ensures that this tensor is simple and non-degenerate. For the example solution with $A^\mu = (R,0,0,0)$ and $B^\mu = (0,R,0,0)$, the only non-zero component is $J^{01} = R^2\omega$, which is the generator of boosts in the internal (0,1) plane. This demonstrates that the specific phase-locked oscillation between T and S^1 corresponds to a constant "internal boost" charge. Different phase relationships and orientations of the amplitude vectors correspond to different conserved Lorentz charges (rotations or boosts), forming a representation of the Lorentz algebra. This directly links the kinematical oscillation of the state vector to the generators of spacetime symmetries, foreshadowing how the algebra of these internal charges may give rise to the Poincaré algebra of an emergent spacetime (Wess & Bagger, 1992).

In conclusion, the Euler–Lagrange equations for the covariant oscillator yield solutions where space and time components are forced into a specific phase relationship by the norm conservation constraint. This phase-locked oscillation, characterized by Minkowski-orthogonal amplitude vectors, is the primary dynamical manifestation of the theory's axioms. It represents a perpetual, harmonic exchange between what we interpret as temporal and spatial aspects of the state, providing a dynamical proto-concept of "passage" and "extension" from which the geometry of spacetime can be constructed in the field-theoretic limit.

Connection to Relativistic Mechanics: Mass as a Geometrical Parameter

Identification with Physical Coordinates

The formalism developed thus far has treated the components of the state vector $\Psi^\mu = (T, S^i)$ as abstract coordinates in an internal Minkowski space. To establish a concrete bridge with established physics, we now posit a fundamental identification. We propose that, upon a suitable choice of units and scaling, these abstract components correspond directly to the coordinates of physical spacetime. Specifically, we make the connection:

$\Psi^\mu \equiv (c\, t(\lambda), x^i(\lambda))$.

Here, t is the physical time coordinate, x^i are the three spatial coordinates, c is the speed of light (a fundamental constant that now emerges as the conversion factor between the temporal scale of the state vector and measured time), and λ is the worldline parameter. This is a pivotal step: it is a physical hypothesis that the dynamics of the conserved state vector, which until now described an abstract internal rotation, is in fact the fundamental description of the trajectory of a physical object in 4D spacetime. This identification is not ad hoc but is strongly motivated by the identical mathematical structure of the theories, a point emphasized in studies of the geometric formulation of mechanics (Rovelli, 2004).

Under this identification, the conserved Minkowski norm of the state vector takes on an immediate and familiar physical meaning. The invariant quantity becomes:

$$Q = \eta_{\mu\nu} \Psi^\mu \Psi^\nu = -(c\, t)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 = -c^2 t^2 + x^2.$$

This is precisely the invariant spacetime interval of special relativity, up to a sign convention. The conservation law $\eta_{\mu\nu} \Psi^\mu \Psi^\nu$ = constant now states that the particle's worldline is confined to a hyperboloid of constant spacetime interval from the origin, which for a massive particle is a timelike hyperbola.

Recovery of the Standard Relativistic Particle Action

Let us examine the core Lagrangian L_0 from Section 3 under this new identification. It becomes:

$$L_0 = -\alpha \sqrt{ - \eta_{\mu\nu} (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda) } = -\alpha \sqrt{ c^2 (dt/d\lambda)^2 - (dx/d\lambda)\cdot(dx/d\lambda) }.$$

We can manipulate this expression by factoring out $(dt/d\lambda)^2$:

$$L_0 = -\alpha\, (dt/d\lambda) \sqrt{ c^2 - (dx/dt)^2 } = -\alpha\, c\, (dt/d\lambda) \sqrt{ 1 - (v^2/c^2) },$$

where $v = dx/dt$ is the coordinate 3-velocity. The action $S = \int L_0\, d\lambda$ is invariant under reparametrization. We are free to choose the parameter λ that is most convenient. The standard choice is to take λ to be the physical time t itself (i.e., $\lambda = t$). With this gauge choice, $dt/d\lambda = 1$, and the Lagrangian simplifies to:

$$L_0(t) = -\alpha\, c \sqrt{1 - v^2/c^2}.$$

The corresponding action is $S = \int L_0(t)\, dt = -\alpha\, c \int \sqrt{1 - v^2/c^2}\, dt$.

To match the well-known action for a free relativistic massive particle, we require the constant α to be:

$$\alpha = m\, c,$$

where m is the inertial mass of the particle. With this identification, the action becomes exactly the Einstein-Infeld-Hoffmann action:

$$S = -m\, c^2 \int \sqrt{1 - v^2/c^2}\, dt.$$

In its more covariant form, using the proper time $d\tau = dt \sqrt{(1 - v^2/c^2)}$, this is:

$S = -m c^2 \int d\tau = -m c \int ds$,

where $ds = c\, d\tau$ is the line element of spacetime. This establishes a complete formal equivalence: the action derived from the dynamics of a conserved state vector is identical to the action for a free relativistic point particle (Landau & Lifshitz, 1975). The constant of motion $Q = \eta_{\mu\nu} \Psi^\mu \Psi^\nu$ is now related to the mass shell condition. From the particle perspective, the canonical momenta $p_\mu = \partial L/\partial(d\Psi^\mu/d\lambda)$ satisfy $\eta^{\mu\nu} p_\mu p_\nu = -m^2 c^2$, which is the standard relativistic dispersion relation.

Mass as a Parameter of State-Space Rotation

This correspondence reveals a profound reinterpretation of the concept of mass. In the state vector formalism, the fundamental constant is the frequency ω (from the oscillator formulation) or the scale parameter $\alpha = m c$. Mass m is not an independent property added to an otherwise massless entity. Instead, it emerges as a geometric parameter that characterizes the scale and nature of the rotation in state space.

Recall the oscillator solution: $\Psi^\mu(\lambda) = A^\mu \cos(\omega\lambda) + B^\mu \sin(\omega\lambda)$. The orthogonality condition $\eta_{\mu\nu} A^\mu B^\nu = 0$ and the constant norm condition $\eta_{\mu\nu} A^\mu A^\nu = \eta_{\mu\nu} B^\mu B^\nu = Q_0$ define the amplitude of this rotation. The frequency ω is related to the mass via the constant α. In the oscillator picture (Section 5), we had $L = (1/2)\, \eta_{\mu\nu} \Psi'^\mu \Psi'^\nu - (1/2)\, \omega^2 \eta_{\mu\nu} \Psi^\mu \Psi^\nu$. The corresponding Hamiltonian constraint in the particle gauge leads to the identification $\omega \sim m c^2 / \hbar$, which is the Compton frequency. This suggests that the mass m is inversely proportional to the period of the fundamental oscillation in state space: a larger mass corresponds to a higher frequency, more "energetic" rotation in the internal space.

Therefore, an object with mass is not simply a static point in spacetime; it is an entity undergoing a perpetual, cyclic evolution in its fundamental state space. Its inertia (resistance to acceleration) can be viewed as a manifestation of the stability of this rotational mode. This perspective resonates with older ideas of "zitterbewegung" in electron theory and more modern concepts of mass generation through cyclical motion in extra dimensions or internal spaces (Hestenes, 1990; Pavšič, 2001). The rest energy $E = m c^2$ can be interpreted as the energy scale associated with this inherent dynamical activity.

Implications for the Field-Theoretic Limit

The identification $\Psi^\mu \equiv (c\, t, x)$ for a single worldline is the first step towards a full field theory. If a single particle's trajectory is described by the rotation of its own state vector, then a system of many particles or a continuum would be described by a field of such state vectors, $\Psi^\mu(x^\nu)$. This creates a conceptual tension: the field Ψ^μ is now a function of the very coordinates (x^ν) that are supposed to be its components. This self-referential structure is the hallmark of emergent geometry.

The resolution is to view the identification $\Psi^\mu(x)$ not as a function from spacetime to itself, but as a map from a base manifold (with coordinates x^ν) to an internal target space. In the simplest case, this map can be trivial ($\Psi^\mu(x) = \delta^\mu_\nu x^\nu$), which corresponds to a flat spacetime. Dynamical distortions of this map, where $\Psi^\mu(x)$ deviates from a linear function, would then represent curvature and gravity. The action for the field $\Psi^\mu(x)$, constructed to be covariant in the base manifold, would generalize the particle action $S = -m c \int ds$. This leads naturally to actions of the form:

$$S[\Psi] = -\Lambda \int d^4x \sqrt{ - \det(\eta_{\mu\nu} \partial_\alpha \Psi^\mu \partial_\beta \Psi^\nu) },$$

which is a Dirac-Born-Infeld type action, a structure known to appear in brane-world scenarios where spacetime itself is a dynamical hypersurface embedded in a higher-dimensional space (Gibbons, 1998). In this grand picture, the mass m of a particle in the effective 4D theory arises from the tension of the worldsheet/brane and the frequency of its fluctuations, directly generalizing the idea of mass as a rotational parameter from the single worldline to the field.

Thus, the link to relativistic mechanics is not merely a consistency check; it is the gateway through which the abstract dynamics of a conserved state vector transmutes into the familiar physics of spacetime and matter, with mass emerging as a fundamental geometric charge.

Physical Interpretation: A Synoptic View

The formalism developed in the previous sections, which derives from the simple axioms of a conserved norm and anti-parallelism, offers a radical reinterpretation of fundamental physical concepts. Rather than being primitive, spacetime coordinates, mass, energy, and motion emerge as derived, phenomenological descriptions of a more fundamental dynamics occurring in an abstract state space. This section provides a concise synopsis of this physical interpretation.

Mass as the Frequency of Internal Rotation

In the covariant oscillator formulation, the fundamental equation is $d^2\Psi^\mu/d\lambda^2 + \omega^2 \Psi^\mu = 0$. The constant ω has dimensions of frequency. Upon identifying the state vector with spacetime coordinates and matching the action to that of a relativistic particle, we find the relation $\alpha = m c$, where α is the proportionality constant in the Lagrangian. In the oscillator picture, this leads to the identification:

$$\omega \sim m c^2 / \hbar.$$

This is the Compton frequency of a particle. Consequently, we interpret mass as the frequency of the inherent rotation of the state vector in its internal Minkowski space. A massive particle is not an inert lump; it is a dynamical entity undergoing a perpetual, cyclical evolution. The higher the mass, the faster this internal "clock" ticks. This resonates with the concept of mass as a measure of an object's intrinsic dynamical activity, a perspective found in models linking inertia to fundamental quantum fluctuations or zitterbewegung (Hestenes, 1990). A massless particle

($\omega=0$) would correspond to a non-oscillatory, linear trajectory in state space, consistent with its null geodesic behavior in spacetime.

Energy as the Rate of State-Space Traversal

In the particle picture, the Hamiltonian derived from the Lagrangian L_0 is a constant of motion. For the simple solution $\Psi^\mu(\lambda) = A^\mu \cos(\omega\lambda) + B^\mu \sin(\omega\lambda)$, the "energy" conjugate to the parameter λ is proportional to $\omega^2 \eta_{\mu\nu} A^\mu A^\nu$. When we fix the gauge $\lambda = t$ (coordinate time), the conserved quantity becomes the relativistic energy $E = \gamma m c^2$, where $\gamma = 1/\sqrt{(1 - v^2/c^2)}$.

From the state-space perspective, this energy is not primary. Instead, it reflects the rate at which the system's state evolves along its trajectory in the state space. In the gauge $\lambda = t$, the speed of evolution is $d\Psi/d\lambda$. The Lorentz factor γ amplifies this rate for a moving particle. Thus, the relativistic energy E is a measure of the "speed" of the internal dynamical process relative to the coordinate time t. This aligns with the idea that energy characterizes the capacity for change. In a cosmological or gravitational context, where the relationship between λ and t can vary, this interpretation suggests that energy is an emergent, observer-dependent quantity that gauges the local "tempo" of the fundamental state evolution (Rovelli, 1991).

Motion as a Tilt in State-Space Trajectory

Consider two fundamental solutions in state space:

1. **At "rest"**: $T(\lambda) = R \cos(\omega\lambda)$, $S^1(\lambda) = R \sin(\omega\lambda)$, $S^2=S^3=0$.
2. **In "motion"**: A solution where the oscillation ellipse is tilted in the (T, S^1) plane, or involves a phase shift between more components.

The first solution, when projected onto spacetime via $\Psi^\mu = (c\ t, x)$, gives $x^1(t) = (R/c) \sin(\omega\ t)$ for small v/c, an oscillatory motion around a mean position. For a pure boost, the solution is a hyperbolic trajectory: $T(\lambda) = R \sinh(\kappa\lambda)$, $S^1(\lambda) = R \cosh(\kappa\lambda)$. Upon gauge-fixing, this yields uniform coordinate velocity $v = c \tanh(\kappa\lambda)$.

Therefore, physical motion in 3D space is interpreted as a specific tilt or orientation of the elliptical/hyperbolic trajectory in the higher-dimensional state space. A state of rest corresponds to a trajectory that is "upright" with respect to the T-axis, while uniform velocity corresponds to a Lorentz-boosted (tilted) version of that trajectory. Accelerated motion corresponds to a trajectory that is not a simple geodesic (straight line or hyperbola) in the state space. This geometric view of motion is reminiscent of the "worldline geometry" perspective in relativity but elevates it to a dynamics in a space of states, not just events (Bohm, 1993).

The Arrow of Time as a Choice of Orientation

The fundamental dynamical equation, $d^2\Psi^\mu/d\lambda^2 + \omega^2 \Psi^\mu = 0$, is time-reversal symmetric with respect to the parameter λ. Its general solution is a linear combination of $\exp(+i\omega\lambda)$ and exp(-iωλ), or sine and cosine. The choice of a specific phase for the oscillation (e.g., $T(\lambda) = R \cos(\omega\lambda)$ rather than $T(\lambda) = R \cos(\omega\lambda + \pi)$) represents a choice of initial condition.

When we identify T with c t, this phase choice becomes critical. A solution where $T(\lambda)$ increases monotonically with λ (e.g., the hyperbolic sine/cosine solution for a timelike norm) defines a direction of evolution in the state space. This chosen orientation, once mapped to the physical time coordinate t, manifests as the psychological or thermodynamic arrow of time. The fundamental theory itself is symmetric; the arrow arises from the specific, low-entropy boundary condition that selects one orientation of the state vector's evolution as "future" (Penrose, 1979). In this view, the flow of time is not a property of spacetime itself but a macroscopic, emergent property of a particular solution to the underlying state-vector dynamics, characterized by a global correlation (phase coherence) in the evolution of its components.

Synthesis: From State Space to Spacetime Phenomenology

The proposed interpretation synthesizes as follows:

- The universe is described by a field $\Psi^\mu(x^\nu)$, a map from an abstract base manifold to an internal Minkowski space.
- A localized particle is a solitonic excitation or a coherent mode of this field, where Ψ^μ executes a localized, cyclical motion.
- The mass (m) of the particle is the frequency (ω) of this cycle.
- Its energy-momentum is the current associated with the translation of this cycle in the base manifold.
- Its worldline in physical spacetime is the shadow (projection) of its higher-dimensional trajectory in the field configuration space.
- The spacetime metric $g_{\alpha\beta}(x)$ is an effective, induced metric derived from the "stretch" of the field map: $g_{\alpha\beta} = \eta_{\mu\nu} \partial\Psi^\mu/\partial x^\alpha \, \partial\Psi^\nu/\partial x^\beta$.
- Gravity arises as the dynamics of this induced metric when the field $\Psi^\mu(x)$ deviates from a trivial, flat embedding.

This framework unifies mechanics and geometry. It suggests that what we perceive as a persistent object moving through time is, at a fundamental level, a stable, rotating pattern in a field. The constancy of the speed of light c is the conversion factor between the scale of the internal rotation (in state-space "meters") and the measured spacetime intervals. This approach shares philosophical underpinnings with process-oriented interpretations of physics and with modern approaches to emergent gravity, such as the "geometrogenesis" of condensed matter analogs (Volovik, 2009) and certain formulations of string theory/M-theory where particles are vibrating strings and spacetime is a derived concept.

In conclusion, the conserved state vector paradigm offers a coherent and minimalist narrative: spacetime, matter, and their dynamics are not separately postulated but are intertwined manifestations of the self-consistency and conservation properties of a single fundamental field.

Canonical Formulation and Conclusions

The Canonical Statement of the Theory

The theoretical framework developed in this work can be summarized by the following core, canonical statement:

The dynamics of physical space and time are described by an invariant-norm Lagrangian of a fundamental state vector; physical evolution corresponds to the Lorentz-rotation of this vector under strict conservation of its Minkowski modulus, with spatial and temporal components manifesting as anti-parallel projections in the state space.

This statement encapsulates the foundational principles, the mathematical structure, and the proposed physical interpretation. It signifies a departure from the conventional view of spacetime as a pre-existing arena, proposing instead that spacetime relations and the dynamics within them are dual manifestations of a more fundamental, algebraic conservation law governing an abstract state vector. This paradigm aligns with broader research programs that seek the origins of geometry in algebraic or informational constraints (Smolin, 2004).

The Mathematical Core: Lagrangian and Symmetries

Mathematically, the theory is anchored in a minimal Lagrangian density. In its most symmetric and fundamental oscillator form, it is expressed as:

$$\mathcal{L} = (1/2)\, \eta_{\mu\nu}\, (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda) - (1/2)\, \omega^2\, \eta_{\mu\nu}\, \Psi^\mu \Psi^\nu.$$

In its worldline-geometric form, equivalent for timelike trajectories, it is:

$$\mathcal{L} = -m\, c\, \sqrt{ - \eta_{\mu\nu}\, (d\Psi^\mu/d\lambda)(d\Psi^\nu/d\lambda) }.$$

Both formulations enforce the conservation of the norm $Q = \eta_{\mu\nu} \Psi^\mu \Psi^\nu$. The first does so through a quadratic potential, making the norm's conservation a consequence of the equations of motion. The second does so through its reparametrization invariance and the associated constraint structure (Henneaux & Teitelboim, 1992).

The theory possesses two paramount symmetries:

1. **Global Internal Lorentz Invariance**: $\Psi^\mu \to \Lambda^\mu{}_\nu \Psi^\nu$, with $\Lambda \in SO(1,3)$. This is the symmetry of the "kinetic" and "potential" terms and reflects the rotational nature of dynamics in state space.

2. **Reparametrization Invariance (for the square-root form)**: $\lambda \to f(\lambda)$. This one-dimensional diffeomorphism invariance signifies that the bookkeeping parameter λ has no intrinsic physical meaning, foregrounding only the relational order of states.

The canonical momenta are $\pi_\mu = \partial\mathcal{L}/\partial(d\Psi^\mu/d\lambda)$, leading to a primary constraint in the square-root formulation: $\eta^{\mu\nu} \pi_\mu \pi_\nu + m^2 c^2 = 0$. This is the mass-shell condition, which upon quantization would yield a Klein-Gordon-like equation in the state space, not in spacetime. The anti-parallelism condition, $S = -c \hat{n} T$, can be implemented as a secondary constraint via Lagrange multipliers, further reducing the physical phase space and linking the internal orientation to observable kinematics.

Physical Evolution as State-Space Rotation

The core dynamical prediction is that the state vector $\Psi^\mu(\lambda)$ evolves not through translation in an external space, but through rotation (a Lorentz transformation) in its own internal space. The general solution to the equations of motion is of the form:

$$\Psi^\mu(\lambda) = M^\mu{}_\nu(\omega\lambda)\, \Psi^\nu(0),$$

where $M^\mu{}_\nu(\omega\lambda)$ is an element of the Lorentz group dependent on the parameter λ and the characteristic frequency ω. For the oscillator, M represents a complex rotation; for the worldline, it is a hyperbolic rotation (boost). Physical processes—perceived as motion, energy transfer, or time evolution—are, in this view, specific sequences of these internal rotations.

The identification of the state vector components with physical coordinates, $\Psi^\mu \equiv (c\, t, x)$, is the crucial "bridging rule" that maps this internal rotation to observable spacetime phenomena. Under this rule, a boost in the (Ψ^0, Ψ^1) plane of state space translates directly into uniform rectilinear motion along the x^1-axis in physical spacetime. The constant speed of light c emerges as the conversion factor that equates a unit of "state-space length" in the temporal direction to a unit of measured time.

Spatial and Temporal Components as Anti-Parallel Projections

The anti-parallelism axiom posits a fixed, rigid relationship: $S^i = -c\, n^i\, T$. In the full dynamical theory, this is understood as a condition that holds for the fundamental mode of a stationary, massive object in its rest frame. More generally, the solutions show that spatial and temporal components are in a fixed phase relationship—quadrature for the free oscillator, strict anti-parallelism when constraints are applied.

This implies that what we measure as space (extension) and time (duration) are not independent entities but are two anti-parallel projections of the same rotating state vector. A change in the temporal projection (ΔT) is always accompanied by a compensatory, anti-aligned change in the spatial projection (ΔS), such that the overall norm is preserved. This provides a novel perspective on the nature of spacetime intervals: the invariant interval $ds^2 = -c^2\, dt^2 + dx^2$ is not a property of a background but a direct measure of the constant magnitude of the evolving state vector. This view resonates with the "3+1" decomposition of spacetime in general relativity, where the lapse function and shift vector describe how the spatial hypersurface is "projected" forward in time, though here the origin is algebraic, not geometric (Arnowitt, Deser, & Misner, 2008).

Conclusions and Future Directions

This article has outlined a program to derive the kinematics and, in prospect, the dynamics of spacetime from the principle of conservation applied to a fundamental state vector. The framework is minimal, covariant, and directly reflects its axiomatic foundations. Key achievements include:

1. Deriving the action for a relativistic particle from the geometry of state-space rotation.
2. Reinterpreting mass as the frequency of internal oscillation.
3. Providing a geometric basis for the phase relationship between space and time.
4. Offering a unified interpretation of motion, energy, and the arrow of time.

The path forward involves several critical research avenues:

- **Field-Theoretic Completion**: The single worldline must be generalized to a field $\Psi^\mu(x^a)$. The primary candidate action is a Dirac-Born-Infeld type:
  $S[\Psi] = -T \int d^4x \sqrt{- \det(\eta_{\mu\nu} \partial_\alpha \Psi^\mu \partial_\beta \Psi^\nu)}$,
  whose perturbations may yield emergent gravitational dynamics (Gibbons, 1998).
- **Quantization**: Applying canonical quantization to the constrained system will determine if the state-space Klein-Gordon equation yields sensible quantum mechanics and connects to the Hilbert space structure of standard quantum theory.
- **Coupling and Matter**: Mechanisms to introduce interactions (gauge fields) and differentiate between various particle sectors (different masses, spins) must be developed, possibly through topological invariants or additional internal symmetries.
- **Connection to Quantum Gravity**: The formalism's emphasis on a fundamental state vector and an emergent metric suggests potential links to approaches like loop quantum gravity (where geometry is quantized) and holographic principles (where dimensionality is not fundamental) (Rovelli, 2004).

In essence, this work proposes that the seemingly disparate concepts of space, time, matter, and motion are unified in the elegant, rotating dance of a conserved state vector. The fabric of reality may be woven not from threads of spacetime, but from the invariant patterns of a deeper, simpler dynamics.

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