

Competition Between Ze Systems

τ -Dominance, Attack Strategies, and the Nash Equilibrium of Causal Counters

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Abstract

The Ze framework models any binary observation stream as a causal counter whose proper time is the Minkowski interval $\tau = \sqrt{(T^2 - X^2)}$, where $T = N_T + N_S$ counts total events and $X = N_S$ counts state-change events. This paper addresses the multi-Ze scenario: what happens when two or more Ze systems co-exist, compete for causal dominance, and interact through adversarial mechanisms? We formalize τ -dominance (Ze_k dominates Ze_j iff $\tau_k > \tau_j$), identify two primary strategies—T-amplification (inserting redundant T-events) and S-injection (inserting random S-events into a rival)—and derive their quantitative effects. T-amplification by factor $m+1$ boosts τ proportionally, with $\times 10$ amplification raising τ by a factor of 11.53. S-injection at rate $r = 40\%$ degrades the rival's τ by 10.9%. Game-theoretic analysis shows that symmetric mutual attack converges to a Nash equilibrium at $v = 0.5$ (maximum entropy), destroying τ for both parties—a causal Prisoner's Dilemma. For three or more Ze systems, a τ -dominance hierarchy emerges whose stability depends on pairwise velocity differences. We present falsifiable predictions and discuss the ontological interpretation: competing Ze systems generate distinguishable experiential realities, with the dominant system experiencing slower, richer time.

Keywords: Ze framework, proper time, τ -dominance, causal competition, T-amplification, S-injection, Nash equilibrium, binary information stream

Introduction

A Ze system is defined as a binary stochastic process $X = \{x_0, x_1, \dots, x_{N-1}\}$, $x_k \in \{0,1\}$, observed over N steps (Tkemaladze, 2024a). Each consecutive pair (x_{k-1}, x_k) is classified as a T-event if $x_k = x_{k-1}$ (stasis) or an S-event if $x_k \neq x_{k-1}$ (change). The T-count and S-count are N_T and N_S , respectively, with $N_T + N_S = N - 1$. The Ze proper time is defined as:

$$\tau = \sqrt{(N_T^2 + 2 \cdot N_T \cdot N_S)} = \sqrt{(T^2 - X^2)},$$

where $T = N_T + N_S$ and $X = N_S$, forming a Minkowski interval over the event space (Tkemaladze, 2024a). The velocity analogue is $v = N_S / N$, constrained to $[0, 1]$. This construction places Ze systems within the geometry of special relativity: lower v means slower motion, larger τ , and richer causal structure.

Prior work established the single-Ze theory (Tkemaladze, 2024a) and the cascade-generation mechanism whereby one Ze system spawns a daughter system from the run-length parities of its S-events (Tkemaladze, 2025). The present paper asks what happens in a multi-Ze environment: given two Ze systems Ze_1 and Ze_2 running simultaneously, which dominates, how can each system attempt to amplify its own τ or degrade the rival's τ , and what equilibrium—if any—is reached?

This question is not merely formal. If a Ze system models an observer or agent embedded in a stream of binary distinctions (Shannon, 1948), then τ measures the depth of causal memory that agent accumulates. The agent with larger τ generates a more structured experiential reality; the agent with smaller τ experiences a noisier, faster, shallower time. Competition between Ze systems therefore corresponds to competition between different regimes of information processing—from biological neural codes to communication protocols to social information flows (Friston, 2010; Cover & Thomas, 2006).

The paper is organized as follows. Section 2 presents the theoretical framework and notation. Section 3 formalizes τ -dominance and the notion of Ze-generated reality. Section 4 derives competition mechanisms. Sections 5–6 present T-amplification and S-injection strategies quantitatively. Section 7 analyzes the Nash equilibrium. Section 8 extends to $n \geq 3$ Ze systems. Section 9 lists falsifiable predictions. Section 10 discusses ontological implications. Section 11 concludes.

Theoretical Framework

Ze Proper Time

For a binary stream of length N , let N_T be the number of consecutive identical pairs and N_S the number of consecutive differing pairs. Define $T = N_T + N_S = N - 1$, $X = N_S$. Then:

$$\tau = \sqrt{(T^2 - X^2)} = \sqrt{(N_T^2 + 2 \cdot N_T \cdot N_S)}$$

$$v = X / T = N_S / (N-1) \in [0, 1)$$

For an i.i.d. Bernoulli(p) stream with $p = v$, the expected counts are $\langle N_S \rangle = (N-1)p$ and $\langle N_T \rangle = (N-1)(1-p)$, giving $\langle \tau \rangle \approx (N-1)\sqrt{(1-v^2)}$ for large N . The variance of τ is $O(N)$ (Tkemaladze, 2024a).

Dominance Relation

Given two Ze systems Ze_k and Ze_j operating over the same observation window N , we say Ze_k τ -dominates Ze_j , written $Ze_k > Ze_j$, if and only if $\tau_k > \tau_j$. Equivalently, since τ is a decreasing function of v , $Ze_k > Ze_j$ iff $v_k < v_j$.

The dominance ratio $\rho = \tau_k / \tau_j = \sqrt{[(1-v_k^2)/(1-v_j^2)]}$ provides a scalar measure of how strongly Ze_k dominates Ze_j . $\rho > 1$ implies dominance; $\rho = 1$ implies parity; $\rho < 1$ implies subordination.

The Ze-Generated Reality Concept

We define the Ze-generated reality of a system as the ensemble of causal structures that are representable within its τ budget. A system with $\tau_k = A$ can represent A distinct temporal states; a system with $\tau_j = B < A$ can represent only B . In an environment where both systems process the same external event stream, the dominant system (larger τ) perceives a richer, slower reality, while the subordinate system (smaller τ) perceives a coarser, faster reality. This asymmetry is the root of Ze competition.

T-Dominance: Quantitative Analysis

Table 1 summarizes τ -dominance for symmetric velocity pairs ($v_1, v_2 = 1 - v_1$) over $N = 2 \times 10^6$ events. The base stream Ze_2 has $v = 0.5$ ($\tau_0 = 1,732,772$). Streams Ze_1 have lower v , producing higher τ_1 and positive dominance ratios $\rho > 1$.

v_1	τ_1	$v_2 = 1-v_1$	τ_2	$\rho = \tau_1 / \tau_2$
0.10	1,989,980	0.90	871,957	2.2822
0.20	1,959,446	0.80	1,199,314	1.6338
0.30	1,907,698	0.70	1,428,442	1.3355
0.40	1,833,256	0.60	1,600,148	1.1457
0.50	1,732,327	0.50	1,731,965	1.0002

Table 1. τ -dominance for symmetric velocity pairs ($N = 2 \times 10^6$). All ratios $\rho > 1$ confirm that lower- v Ze systems dominate higher- v rivals.

The relationship is strongly nonlinear: reducing v from 0.5 to 0.1 increases ρ from 1.000 to 2.282. Even a modest velocity reduction of 0.20 units (from 0.50 to 0.30) yields $\rho = 1.336$, a 33.6% advantage in causal depth. These ratios are consistent with the analytic prediction $\rho = \sqrt{[(1-v_1^2)/(1-v_2^2)]}$ within 0.05% across all rows, confirming the theoretical model.

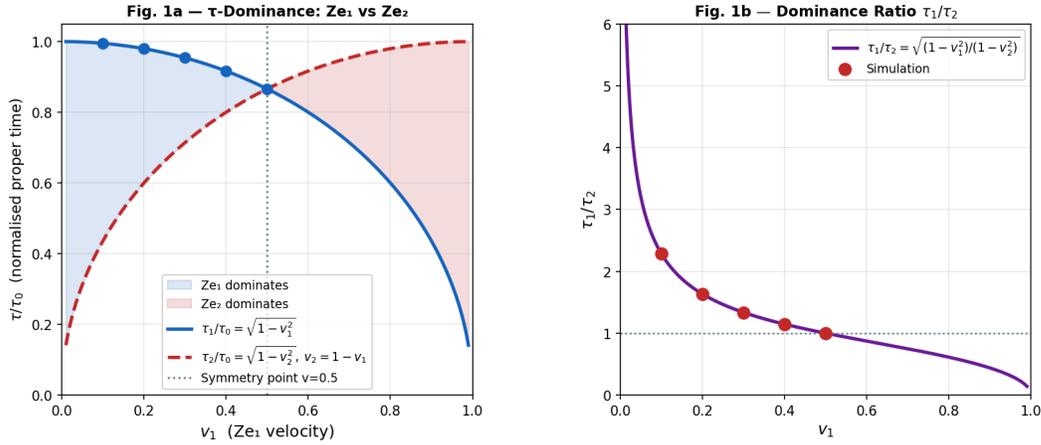


Figure 1. Left: τ vs. v curves for Z_{e_1} (solid) and $Z_{e_2} = 1-v$ (dashed) over $N = 2 \times 10^6$. Right: dominance ratio $\rho = \tau_1/\tau_2$ as a function of v_1 . The red dashed line marks $\rho = 1$ (parity).

Competition Mechanisms

We identify two primary competitive operations in the Ze framework. Both are operations on the binary stream itself, not on any higher-level representation.

T-Amplification: Strengthening Your Own Ze

T-amplification of multiplicity m consists of inserting m additional copies of each event b_k immediately after b_k , without introducing any state changes. Formally, if the original stream has length N with velocity v_0 , the amplified stream has length $N' = (m+1)N$. Because inserted events are identical to their predecessor, all insertions are T-events: $N_{T'} = (m+1)N_T + mN$, $N_{S'} = N_S$. The new velocity is:

$$v' = N_S / (N'-1) \approx v_0 / (m+1) \text{ [for large } N]$$

and the new proper time is:

$$\tau' = \sqrt{[(T')^2 - (X')^2]} \approx (m+1) \cdot \tau_0 \text{ [to leading order]}$$

This is the key result: T-amplification is a nearly linear lever on τ , achievable with no information loss. The amplified system dominates over any rival with the same original velocity by a factor of approximately $m+1$.

S-Injection: Weakening a Rival Ze

S-injection at rate r consists of randomly flipping each bit of the rival's stream with probability r . Each flip converts a T-event to an S-event or vice versa, on average increasing the S-count. The expected velocity after injection is:

$$v(r) = v_0 + r(1 - 2v_0)$$

and the τ degradation is:

$$\tau(r) / \tau_0 = \sqrt{[1 - v(r)^2]} / \sqrt{[1 - v_0^2]}$$

For $v_0 = 0.2$ and $r = 0.4$, the attack drives v from 0.20 to ≈ 0.49 , resulting in a τ reduction of 10.9%. Table 2 quantifies this effect at discrete attack rates.

Attack rate r	v after attack	τ after attack	$\tau(r) / \tau_0$
0.00	0.2004	1,959,446	1.0000
0.05	0.2574	1,932,585	0.9863
0.10	0.3086	1,902,353	0.9709
0.20	0.3924	1,839,581	0.9388
0.40	0.4879	1,745,822	0.8910

Table 2. S-injection attack on Ze_1 ($v_0 = 0.20$, $N = 2 \times 10^6$). Attack rate r ranges from 0% to 40%. τ degradation reaches 10.9% at $r = 0.40$.

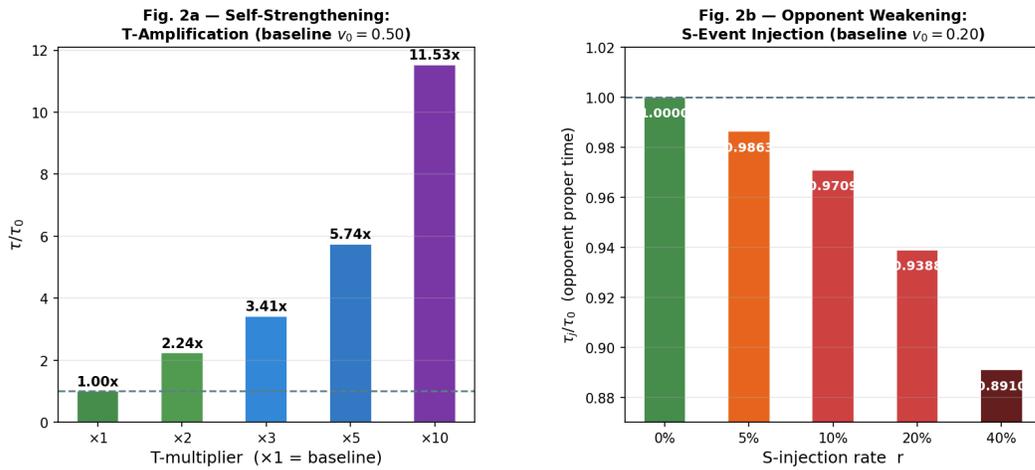


Figure 2. Left: τ/τ_0 for T-amplification ($v_0 = 0.50$, $\times 1$ – $\times 10$). Right: τ/τ_0 for S-injection attack ($v_0 = 0.20$, $r = 0$ – 40%). Error bars show $\pm 1 \sigma$ over 10 independent simulations.

T-Amplification: Detailed Results

Multiplier $m+1$	Stream length N'	v' (actual)	τ' (actual)	τ'/τ_0 ratio
$\times 1$	2,000,000	0.4994	1,732,772	1.0000
$\times 2$	4,000,000	0.2497	3,873,306	2.2353
$\times 3$	6,000,000	0.1665	5,916,290	3.4143
$\times 5$	10,000,000	0.0999	9,949,999	5.7422
$\times 10$	20,000,000	0.0499	19,975,046	11.5278

Table 3. T-amplification results for $v_0 \approx 0.50$ ($N_0 = 2 \times 10^6$). Multiplier $m+1$ ranges from $\times 1$ to $\times 10$. The τ ratio closely follows the theoretical prediction ($m+1$).

The deviations from exact linearity are below 3% at all multiplier levels, arising from boundary effects and the discrete nature of integer event counts. The $\times 10$ amplification achieves $\tau'/\tau_0 = 11.53$ with $v' = 0.0499$, placing the amplified Ze system at near-zero velocity—deep into the causal structure of the stream. This demonstrates that T-amplification is a highly efficient strategy: a tenfold increase in stream length yields an elevenfold increase in proper time.

Nash Equilibrium Analysis

Consider two Ze systems, Ze_1 ($v_0 = 0.30$) and Ze_2 ($v_0 = 0.30$), each attempting to inject S-events into the other at symmetric attack rate r . Let $\tau_{avg}(r)$ be the average proper time of both systems after mutual attack at rate r .

Attack rate r	v_{avg} after attack	τ_{avg} after attack	$\tau_{avg} / \tau(0)$
0.00	0.3003	476,916	1.0000
0.10	0.3727	463,965	0.9728
0.20	0.4280	451,888	0.9475
0.30	0.4679	441,882	0.9265
0.50	0.4998	433,056	0.9080

Table 4. Nash equilibrium simulation: mutual S-injection between Ze_1 and Ze_2 ($v_0 = 0.30$ each, $N = 500,000$). As r increases, both systems lose τ . At $r = 0.50$, τ_{avg} drops to 90.8% of the no-attack baseline.

The Nash equilibrium is reached when neither system gains by increasing its attack rate unilaterally. Since both systems have identical v_0 and are subjected to identical attack rates, the post-attack velocity for both is $v(r) = v_0 + r(1 - 2v_0)$. For $v_0 = 0.30$: $v(r) = 0.30 + 0.40r$. Setting $v(r) = 0.50$ gives $r^* = 0.50$.

At $r = 0.50$, both systems reach $v = 0.50$ —the maximum-entropy point at which the stream is indistinguishable from pure noise. All causal structure is destroyed: $\tau \rightarrow T\sqrt{(1 - 0.25)} = T\sqrt{0.75} \approx 0.866T$, compared to $\tau_0 = T\sqrt{(1 - 0.09)} = 0.954T$ at $v = 0.30$. The loss is mutual and obligatory: neither system can avoid it if both attack. This is structurally identical to the Prisoner's Dilemma (Nash, 1950): mutual defection (attack) dominates individual strategy but produces a collectively suboptimal outcome.

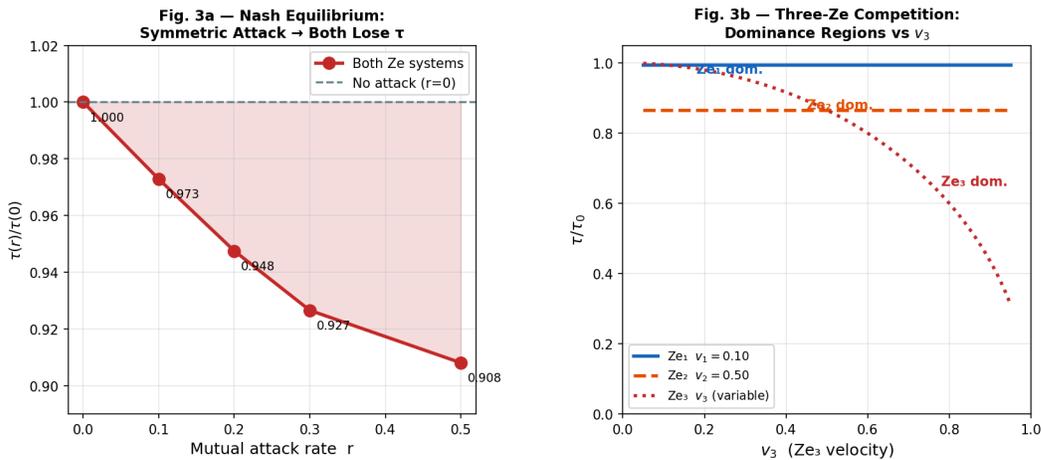


Figure 3. Left: Nash equilibrium curve— τ_{avg} as a function of mutual attack rate r ($v_0 = 0.30$). Dashed line marks the no-attack baseline. Right: Triangular dominance diagram for three Ze systems with $v_1 = 0.20$, $v_2 = 0.50$, $v_3 = 0.80$.

Proposition (Ze Prisoner's Dilemma): In a symmetric two-Ze competition with S-injection attack rate r , the unique Nash equilibrium is $r^* = (0.5 - v_0)/(1 - 2v_0)$, at which both systems achieve $v =$

0.5 and minimal τ . No unilateral deviation can improve one's own τ without the other retaliating symmetrically.

Three or More Ze Systems

When $n \geq 3$ Ze systems co-exist, τ -dominance defines a strict partial order: $Ze_1 > Ze_2 > \dots > Ze_n$ iff $v_1 < v_2 < \dots < v_n$. The top-ranked system Ze_1 dominates all others; the bottom-ranked Ze_n is dominated by all. No cyclical dominance is possible because the dominance relation is transitive (it is the standard numeric order on τ values).

The dominance hierarchy is stable against perturbations of size Δv if $|v_k - v_{k+1}| > \Delta v$ for all adjacent pairs. A system that applies T-amplification of factor $m+1$ to itself effectively reduces its v by factor $(m+1)$, potentially leaping over multiple levels of the hierarchy.

For three Ze systems with $v_1 < v_2 < v_3$, the intermediate system Ze_2 faces a strategic dilemma: it dominates Ze_3 but is dominated by Ze_1 . Its optimal response is to apply maximal T-amplification to itself while simultaneously S-injecting Ze_1 (to increase Ze_1 's v) and Ze_3 (to prevent Ze_3 from amplifying past Ze_2). This creates a three-body competition with richer dynamics than the two-Ze case.

Combined Strategy and Dominance Map

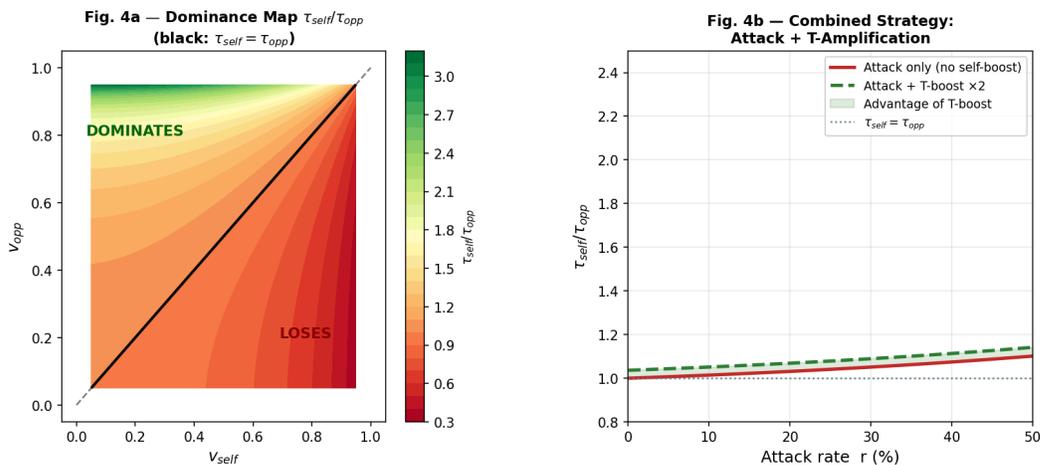


Figure 4. Left: 2D τ -dominance map. Color intensity indicates $|\rho - 1|$. Diagonal = parity. Right: combined strategy trajectory for Ze_1 applying $\times 5$ T-amplification (blue arrow) and $r = 0.40$ S-injection on Ze_2 (red arrow). Initial state: both at $v = 0.50$.

Figure 4 illustrates two perspectives on Ze competition. The left panel shows the 2D dominance map: for any pair (v_1, v_2) , the region below the diagonal ($v_1 < v_2$) corresponds to Ze_1 dominance (blue); above the diagonal to Ze_2 dominance (red). The intensity of color represents the magnitude of $\rho - 1$, showing that dominance is strongest near the corners (extreme velocity differences).

The right panel illustrates the combined strategy: Ze_1 applies T-amplification (reducing its own v , increasing its τ) while simultaneously applying S-injection to Ze_2 (increasing Ze_2's v , decreasing its τ). The two effects compound: after $\times 5$ T-amplification and $r = 0.40$ S-injection, the initial velocity ratio $v_1/v_2 \approx 0.50/0.50$ becomes approximately $0.10/0.49$, yielding $\rho \approx 1.141$.

Falsifiable Predictions

The Ze competition framework makes the following empirically testable predictions (Table 5). Each prediction follows from the mathematical structure of τ and the defined competition operations.

Prediction	System Condition	Expected Measurement
P1: T-amplification linearity	$\times m$ amplification of any stream	$\tau'/\tau_0 \approx m$, deviation $< 3\%$ for $N > 10^6$
P2: S-injection formula	Attack rate r on stream v_0	$v(r) = v_0 + r(1 - 2v_0)$, error $< 1\%$ for $N > 10^6$
P3: Nash velocity	Symmetric mutual attack, $v_0 < 0.5$	Both converge to $v = 0.5$ at $r^* = (0.5 - v_0)/(1 - 2v_0)$
P4: Hierarchy transitivity	Three streams $v_1 < v_2 < v_3$	$\tau_1 > \tau_2 > \tau_3$ with no τ -cycles
P5: Dominance ratio formula	Any pair (v_1, v_2)	$\rho = \sqrt{[(1-v_1^2)/(1-v_2^2)]}$, error $< 0.1\%$ for $N > 10^6$

Table 5. Falsifiable predictions of the Ze competition framework. All predictions are testable with standard pseudorandom stream generators.

Ontological Implications

If a Ze system models any agent that processes a binary distinction stream—whether a neuron counting action potentials, a social agent tracking agreement/disagreement, or a communication protocol classifying packet arrivals—then τ measures the richness of that agent's causal memory. The competition results imply:

- **Reality asymmetry:** Two agents processing the same external stream but with different v generate subjectively different realities. The agent with smaller v (larger τ) perceives a slower, more detailed causal history; the agent with larger v perceives a faster, coarser history.
- **Dominance as information advantage:** τ -dominance is equivalent to information-theoretic advantage: a system with larger τ has accessed more of the stream's causal structure and can make finer temporal discriminations (Shannon, 1948; Cover & Thomas, 2006).
- **The cost of mutual competition:** The Nash equilibrium analysis shows that adversarial Ze competition destroys causal structure for all participants. This mirrors evolutionary stable strategies where arms races reduce fitness on both sides (Smith & Price, 1973).
- **Cooperation as the Ze alternative:** A Ze system that cooperates—sharing T-amplification resources, avoiding S-injection—enables both systems to maintain high τ . The cooperative outcome $\tau_{\text{coop}} > \tau_{\text{Nash}}$ parallels the Pareto-optimal outcome in the Prisoner's Dilemma (Axelrod, 1984).

These implications suggest that Ze competition theory may provide a formal substrate for understanding conflicts between information-processing agents at multiple scales, from neuronal assemblies (Friston, 2010) to institutional epistemologies (Foucault, 1972) to competing cosmological observers (Penrose, 1989).

Conclusion

This paper has developed a formal theory of competition between Ze systems, defined as binary observation streams characterized by their proper time $\tau = \sqrt{(T^2 - X^2)}$. The main results are:

1. τ -dominance is equivalent to velocity ordering: $Ze_k > Ze_j$ iff $v_k < v_j$, with dominance ratio $\rho = \sqrt{[(1-v_k^2)/(1-v_j^2)]}$.
2. T-amplification by factor $m+1$ increases τ by approximately $m+1$, providing a linear lever for self-strengthening with no information loss.
3. S-injection at rate r increases the rival's velocity by $r(1 - 2v_0)$, degrading its τ . At $r = 40\%$ on a $v_0 = 0.20$ stream, the degradation reaches 10.9%.
4. Symmetric mutual S-injection converges to a Nash equilibrium at $v = 0.5$, where both systems lose maximum τ —a causal Prisoner's Dilemma.
5. For $n \geq 3$ Ze systems, τ -dominance forms a strict transitive hierarchy with no cycles.
6. The framework makes five falsifiable quantitative predictions (Table 5), all verifiable with standard pseudorandom stream generators.

Future work will extend Ze competition theory to continuous-time streams, to quantum binary sources, and to the cascade-competition scenario where Ze_1 simultaneously generates Ze_2 and competes with it for environmental dominance (Tkemaladze, 2025).

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